Poster Session

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ABSTRACTS

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Modeling Invasive Species Dynamics

We employ Evolutionary Algorithms to select the most suitable model of biological invasions from a suite of diffusion-reaction models. The models incorporate biologically realistic phenomena such as diffusion, advection, habitat heterogeneity, long and short distance transport and density-dependent growth. We generalize the reaction component with biologically meaningful parameters to describe all types of growth. In particular, we include the more recent Allee effect into this reaction term. Depending on parameter values, this new model can describe a suite of 11 different growth models.

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How to Turn Math Duds into Math Studs

This NSF-funded project introduced a web-based curriculum in the developmental algebra course (Intermediate Algebra) in the fall semester of 2002 at East Central University. The ALEKS (Assessment and LEarning in Knowledge Spaces) curriculum is an artificial intelligence-based system for individualized mathematics learning available via the World Wide Web. The ALEKS system provides assessment to determine the student's current knowledge state, and it provides instruction at any given time only on topics which the student is ready to learn. Thus, it permits true individual progression through the mathematics course. Students can access ALEKS from any computer with an Internet connection. Results of the on-going research component of the project will be presented. The study examined the effectiveness of the ALEKS curriculum in teaching developmental algebra. Data analyses involved data that was collected since the inception of the project in the fall semester of 2002 through the spring semester of 2003. Quantitative data were analyzed in a comparison of students' success rates in subsequent mathematics courses (College Algebra and Survey of Math) beyond Intermediate Algebra. Success rates of ALEKS students were compared with those of students who either took developmental algebra in the traditional lecture format or who did not take a developmental

algebra course at all. In addition, current pass rates in Intermediate Algebra were compared to pass rates in the developmental algebra courses taught before ALEKS was adopted in 2002. Qualitative data from surveys and personal interviews were examined in order to evaluate students' attitudes toward the new curriculum as well as certain behavioral patterns.

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Modeling Mullins-Sekerka Flow in Three Space Dimensions

The Mullins-Sekerka problem involves modeling a binary material with two stable concentration phases. A coarsening process occurs, and large particles grow while the smaller particles eventually dissolve. Single particles become spherical. My joint research with Dr. Peter Bates describes this process in three-dimensions using boundary integrals, which can then be solved numerically. The evolution of the particles is simulated with the help of a supercomputer.

Numerical tasks in my research include discretizing three dimensional surfaces, overcoming node bunching, and dealing with topology changes in particles. Prohibitive run time and memory requirements have been dealt with in part by a particle (node)-cluster technique and by utilizing particle symmetry.

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Non-linear Throughput Time Models for a Supply Chain

A fundamental problem in developing production supply chain models has been their inability to reflect the nonlinear dependency between workload and through-put times. This typically leads to significant differences between planned and realized through-put times. In the proposed model of the supply chain, the non-linearity in through-put time is captured using a finite set of steady state and transient state output distributions for every days input. The flow of material across the supply chain is modeled as a linear programming (LP) problem with decision variables controlling the various release points of the supply network.

Every input to a manufacturing unit is assigned a finite set of output

distributions depending on its size and the state of the supply chain. This turns the LP problem into a mixed binary integer-programming problem; such programs are known for their exponentially long run time. To improve the run time of the model, genetic algorithm (GA) is employed with the LP formulation in identifying a good set of inputs and the corresponding output distributions in a reasonable amount of time. This hybrid GA-LP model captures the non-linearity of the through-put time and facilitates decision-making in a reasonable amount of time.

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Star-operations Induced by Overrings

Let D be an integral domain with quotient field K. A star-operation $\star$ on D is a closure operation $A \longmapsto A^\star$ on the set of nonzero fractional ideals, $F(D)$, of D satisfying the properties: $(xD)^\star = xD$ and $(xA)^\star = xA^\star$ for all $x \in K^*$ and $A \in F(D)$. Let $\mathcal{S}$ be a multiplicatively closed set of ideals of D. For $A \in F(D)$ define $A_\mathcal{S} = \{x \in K \mid xI \subseteq A, \text{ for some } I \in \mathcal{S}\}$. Then $D_\mathcal{S}$ is an overring of D and $A_\mathcal{S}$ is a fractional ideal of $D_\mathcal{S}$. Let $\mathcal{S}$ be a multiplicative set of finitely generated nonzero ideals of D and $A \in F(D)$; then the map $A \longmapsto A_\mathcal{S}$ is a finite character star-operation if and only if for each $I \in \mathcal{S}$, $I_v = D$. Here v is the v-operation defined by the map $A \longmapsto A_v = (A^{-1})^{-1}$ for all $A \in F(D)$, where $A^{-1} = \{x \in K \mid xA \subseteq D\}$. I give an example to show that this result is not true if the ideals are not assumed to be finitely generated. In general, the map $A \longmapsto A_\mathcal{S}$ is a star-operation if and only if $\bar{\mathcal{S}}$, the saturation of $\mathcal{S}$, is a localizing GV-system. I also discuss star-operations given of the form $A \longmapsto \cap AD_\alpha$, where $D = \cap D_\alpha$.

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Singular Values and Schmidt Pairs of Composition Operators on the Hardy Space

Suppose φ is a holomorphic self-map of the open unit disc D, that is,

$\varphi(D) \subseteq D$. A composition operator is defined by

$$C_\varphi f = f \circ \varphi, \quad \text{for all } f \text{ holomorphic on } D.$$

On the classical Hardy space we compute the singular values $\{s_k(C_\varphi)\}$ and corresponding Schmidt vectors $\{f_k, g_k\}$ (or singular vectors) of a composition operator C_φ with $\varphi(z) = az + b$ such that $|a| + |b| < 1$. This leads to the singular value decomposition of the operator, that is,

$$C_\varphi f = \sum_{k=0}^{\infty} s_k \langle f, f_k \rangle g_k.$$

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On the Degree of Local Permutation Polynomials

Every Latin square of prime or prime power order s corresponds to a polynomial in 2 variables over the finite field on s elements, called the local permutation polynomial. What characterizes this polynomial is that its restrictions to one variable are permutations. We discuss the general form of local permutation polynomials and prove that their total degree is at most $2s-4$, and that this bound is sharp. We also show that the degree of the local permutation polynomial for Latin squares having a particular form is at most $s-2$. This implies that circulant Latin squares of prime order p correspond to local permutation polynomials having degree at most $p-2$. Finally, we discuss a special case of circulant Latin squares whose local permutation polynomial is linear in both variables.

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Evacuation and a Geometric Construction for Fibonacci Tableaux

Tableaux have long been used to study combinatorial properties of permutations and multiset permutations. Discovered independently by Robinson and Schensted, and generalized by Knuth, the Robinson-Schensted correspondence has provided a fundamental tool for relating permutations to tableaux.

Schützenberger defined a process called evacuation on standard tableaux which gives a relationship between the pair of tableaux (P, Q) resulting from the Schensted correspondence. Viennot gave a geometric construction for the Schensted correspondence, and Fomin described a generalization of the correspondence which provides an immediate bijection between a permutation and pairs of chains in Young's lattice. In 1975, Stanley defined a Fibonacci lattice and in 1988 he introduced the idea of a differential poset which gave rise to a second Fibonacci lattice called $Z(r)$. Roby gave an insertion algorithm, analagous to the Schensted correspondence, for mapping a permutation to a pair of Fibonacci tableaux. The main results of my research are to give an evacuation algorithm for the Fibonacci tableaux that is analogous to the evacuation algorithm on Young tableaux and to describe a geometric construction for the Fibonacci tableaux that is similar to Viennot's geometric construction for Young tableaux.

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Interactive Modules for Introduction to Linear Algebra

This poster will outline the development of interactive modules for Introduction to Linear Algebra course at IUP. This course is typically taken by mathematics, science, and economics majors. Because of students diverse background, it is necessary to develop alternate instructional methods so that students can visualize some of abstract concepts. The modules are programmed in MATLAB and contain interactive features. Students can explore the concepts by trying different data. The goal of this poster is to seek suggestions and comments for further improvement.

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Those Power-Hungry Mathematicians

In a small liberal arts institution, there can be multiple challenges to incorporate software and equipment in the classroom because there are limited funds, human resources, and time. In addition, with only one tenure-track faculty member, building a program, assessing it, and incorporating change can be a daunting task. Furthermore, a mathematics educator is compelled

to incorporate higher order thinking skills into the mathematics curriculum to ensure that the students are receiving the best possible education. To accomplish all of this, a student learning proposal was written and granted by our Faculty Development Committee using Bush Foundation resources. This poster will discuss the process of implementing the student learning proposal to enhance my Differential Equations class and the results of that implementation. As a laptop campus, we were able to obtain a copy of Maple for each student machine. Careful consideration of Bloom's taxonomy was made when assigning a series projects using the computer algebra system so that increased higher level thinking could be assessed. A series of student evaluations and scoring rubrics were employed to supply assessment data. Overall, the Maple-based projects increased student confidence, provided the opportunity to improve performance throughout the series of projects, and resulted in student learning at the higher levels of Bloom's taxonomy.

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Estimating Linear Functionals of Indirectly Observed Input Functions

The input of a system will be discussed when observations on the output are given with known transformation. To recover the information from the output this transformation must be inverted. Since the inverse problems are often ill-posed, a regularized inverse will be considered. Also, we will use the fact that the input function in the Hilbert space is expanded in an orthonormal basis and its Fourier coefficient will be estimated. Since the traditional estimator of the coefficient is in general not efficient in the Hájek-LeCam sense but $\sqrt{n}$-consistent, it can be improved. The theory will be presented in a self-contained manner. This means that the theorem on improving the $\sqrt{n}$-consistent estimator will be derived.

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Some Lack-of-Fit Tests based on Martingale Residuals for the Censored Two-sample Accelerated Life Model

In Survival Analysis, one of the important problems is the comparison of the lifetimes of two groups. This can occur, for example, when two different treatments are given to the two groups, or new treatments are given to an

existing one. When studying possibly-censored lifetimes from two groups, the proportional hazards model has been widely used. The two-sample accelerated life model provides an alternative to the proportional hazard model. It has the simple interpretation that treatment accelerates or decelerates the lifetime by a scale factor. We consider two processes: Observed process and Simulated process. For comparison, we can plot the observed process along with a number of simulated processes. These comparisons enable us to assess objectively how unusual the pattern of the observed process is by looking at the behavior of both. This leads to the construction of lack-of-fit tests.

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Homogenization of a Darcy-Stokes System Modeling Flow in Vuggy Porous Media

We model single phase, incompressible, viscous fluid flow in a porous medium containing cavities called vugs. Vugs are essentially inclusions in the medium which are large relative to the pore size. We model the vuggy medium on the microscopic scale using Stokes equations within the vugs, Darcy's law within the porous rock, and a Beavers-Joseph-Saffman boundary condition on the interface between the two regions. Assuming periodicity of the medium, we obtain a macroscopic Darcy's law through a two-scale homogenization limit. In addition, we formulate a mixed Discontinuous Galerkin/Raviart-Thomas numerical method and obtain error estimates. We plan to have preliminary numerical results as well.

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Undergraduate Mathematics Students in a Transitional Course: Their Struggles and Strategies

The transition from computational mathematics to theoretical mathematics tends to be difficult for many undergraduate mathematics students. As a result, many universities have developed what is known as a "Transitional Course" designed to teach students how to write valid mathematical proofs, assisting them in this transition. For this study, I conducted task-based interviews with students in such a course. I took a qualitative look at some of

the struggles students had when trying to write mathematical proofs and what strategies they found to be successful. I found that the students in my study had great difficulties dealing with the notion of infinity. Mathematical notation was also a great difficulty for many students, especially the idea of keeping certain notation arbitrary within proofs. These students also demonstrated difficulties with understanding the structure of mathematical statements and with deviating from the structure of direct proofs. Common strategies these students used included making mathematical connections, using numerical examples, and considering norms of structured education. Symbolic logic also proved to be a useful tool for many of these students. Symbolic logic not only aided many students in constructing valid proofs, but also was successfully used by students for the process of evaluating proofs.

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Band Structure in Bose-Einstein Condensates in Periodic Lattices

Since their experimental discovery in 1995, Bose-Einstein condensates (BECs) have generated considerable excitement in the physics community both because their study allows one to explore new regimes of fundamental physics and because of their eventual engineering applications. The macroscopic behavior of BECs at 0 Kelvin is modeled by a nonlinear Schrodinger equation (NLS) in the presence of an external potential. When this potential is spatially periodic (e.g., due to an optical lattice, which may be created using counter-propagating laser beams), the BEC spectrum exhibits a band structure (spatial resonance structure).

In this poster, I discuss the use of Hamiltonian perturbation theory and supporting numerical simulations to study coherent structures in BECs and their concomitant band structure in detail. This perturbative approach relies on the elliptic function structure of solutions in the absence of an external potential. I will also discuss recent generalizations of this theory to coupled BECs, which are described by coupled Nonlinear Schrodinger equations. My work on single-species condensates is joint with Predrag Cvitanovic and that on interacting condensates is joint with Boris Malomed and Panos Kevrekidis.

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When "Good" Rings Go "Bad", They Go "Very Bad"

More formally, **Associated Graded Rings Of Complete Intersection Numerical Semigroup Rings**. Often when we pass ("go") from a local ring (R, m) to its associated graded ring $gr_m(R)$ we note that some properties of R do not hold in $gr_m(R)$. We will examine the associated graded ring of $R = k[t^a, t^b, t^c]_m$ where m is the homogeneous maximal ideal. In particular, when R is a complete intersection ("good") but $gr_m(R)$ is not a complete intersection and is also not Cohen Macaulay ("bad"), then $gr_m(R)$ is also not Buchsbaum ("very bad").

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Service Learning in Mathematics for Pre-service Elementary Teachers

In a typical mathematics course for elementary teachers, the content can be criticized as abstract, simplistic, or overly pedagogical, depending on who is doing the critiquing. In a recent course for elementary teachers, I attempted to bridge the gap between abstract content and actual teaching with a service learning experience for the students. The students spent two half-hour sessions each week working with elementary-age students in mathematics and looking for connections between our course material and their tutoring. Students found the material, particularly on multiple base arithmetic, more worthwhile after their service learning experience. Other successes, as well as failures, will be presented. Future directions for the service-learning component of this course will also be discussed.